Parameter Estimation of Hysteresis Elements Using Harmonic Input

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NOMENCLATURE

c	Viscous Damping
F	Force
FRF	Frequency Response Function
f	Frequency
k	Stiffness
m	Mass
t	Time
ω	Angular frequency
x	Displacement
Z	Impedance

ABSTRACT

The industrial demand on good dynamical simulation models is increasing. Since most structures show some form of nonlinear behavior, linear models are not good enough to predict the true dynamical behavior. Hysteresis is a highly nonlinear phenomenon which occurs in for example dampers and mechanical joints.

This paper presents a method for parameter estimation on nonlinear systems under harmonic excitation. By using the principle of harmonic balance or multi harmonic balance a theoretical frequency response function of the studied system can be estimated. This frequency response function can, in conjunction with measured nonlinear transfer functions, be used to make parameter estimations of the nonlinearity present in the system. A major benefit using this method is the ability to use arbitrary nonlinear functions. This means that the method can be applied to nonlinear systems with memory, for instance systems with hysteresis effects. The method is applied to both simulated systems and an experimental test rig.

1. INTRODUCTION

The overall aim with parameter estimation is to find suitable parameters to a mathematical model, based on measurements of the inputs and outputs of a system. The mathematical model can, for instance, be based on a beforehand known model of the studied system. This is known as parametric modelling, which is the method used in this paper.

Due to the benefit of being able to use arbitrary nonlinear functions in the parameter estimation procedure proposed in this paper, the focus is on systems with hysteresis effects. Many mechanical systems exhibit hysteresis effects when subjected to dynamic loading. An interesting fact concerning hysteresis systems is that the nonlinear restoring force isn't only dependent on the instantaneous deformation but also on the past history deformation. As a result the hysteresis nonlinearity has a memory function which makes the system more difficult to model and analyze than general zero-memory nonlinear systems. However, modeling and identification of this type of systems is of great importance in for example response prediction of mechanical systems and structural control.

The following chapters contain the theoretical development of the parameter estimation procedure, followed by a simulated example and the method is also applied to an experimental test rig.

2. THEORY

By exciting a nonlinear system with either a low or a high force, depending on the type of nonlinearity, the frequency response function of the underlying linear system can be estimated. This FRF can be used as basis to calculate an analytical nonlinear FRF of the studied system. The basic principle of this will be described by using a single degree of freedom system with an arbitrary nonlinearity, see figure 2.1.

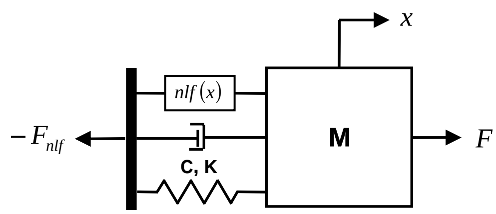

Figure 2.1: Single degree of freedom system with an arbitrary nonlinearity

The equation of motion for this system is:

$$m\ddot{x} + c\dot{x} + kx = F - F_{nlf} \quad (1)$$

F_{nlf} = displacement dependent force due to the nonlinearity.

$$F(t) = F_0 e^{j\omega t} \quad (2)$$

When the system above is excited with a pure sinusoidal force, according to Equation (2), the system response will contain higher harmonics due to the nonlinearity. The displacement can be expressed as a Fourier series according to equation (3).

$$x(t) = \sum_{k=1}^{\infty} x_k e^{jk\omega t} \quad (3)$$

Differentiation of this equations yields:

$$\dot{x}(t)=\sum_{k=1}^{\infty} jk\omega x_k e^{jk\omega t} \tag{4}$$

$$\ddot{x}(t)=-\sum_{k=1}^{\infty} k^2\omega^2 x_k e^{jk\omega t} \tag{5}$$

Putting equations (3- 5) into the equation of motion (1) and considering for example three harmonics (1 3 5), the system of equations become:

$$\begin{aligned}
&-mx_1\omega^2 e^{j\omega t}+jcx_1\omega^2 e^{j\omega t}+kx_1 e^{j\omega t}-F_0 e^{j\omega t}+F_{nl1}e^{j\omega t}=0\\
&-9mx_3\omega^2 e^{j3\omega t}+3jcx_3\omega^2 e^{j3\omega t}+kx_3 e^{j3\omega t}+F_{nl3}e^{j3\omega t}=0\\
&-25mx_5\omega^2 e^{j5\omega t}+5jcx_5\omega^2 e^{j5\omega t}+kx_5 e^{j5\omega t}+F_{nl5}e^{j5\omega t}=0
\end{aligned} \tag{6}$$

Equation (6) can be rewritten in more compact form:

$$\begin{aligned}
&\left(-m\omega^2+jc\omega+k\right)x_1-F_0+F_{nl1}=0\\
&\left(-9m\omega^2+3jc\omega+k\right)x_3+F_{nl3}=0\\
&\left(-25m\omega^2+5jc\omega+k\right)x_5+F_{nl5}=0
\end{aligned} \tag{7}$$

As is evident the bracketed expressions contain the impedance of the underlying linear system, therefore equation (7) can be written as:

$$\begin{aligned}
&Z_1x_1-F_0+F_{nl1}=0\\
&Z_3x_3+F_{nl3}=0\\
&Z_5x_5+F_{nl5}=0
\end{aligned} \tag{8}$$

By using the FRF of the underlying linear system, the impedance is estimated and an analytical nonlinear FRF can be calculated using the method of harmonic balance. This however presumes that the nonlinearity in the system is known. If the nonlinearity is not known, it can be estimated by fitting the analytical nonlinear FRF to a measured or simulated nonlinear FRF. The parameters which define the nonlinear model can then be estimated by the method described in figure 2.2.

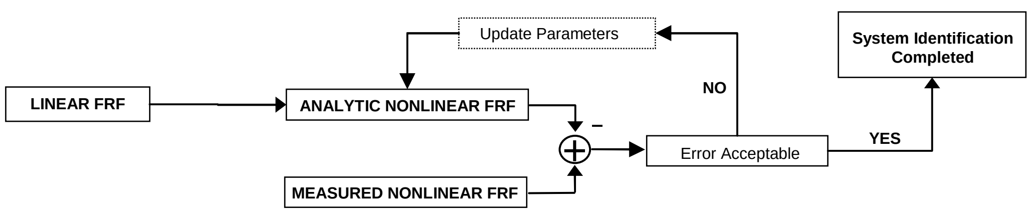

Figure 2.2: Flowchart of the parameter estimation procedure

The cost function used as error estimate is defined as

$$\varepsilon=\left\|H_{measured\_nonlinear}-H_{analytic\_nonlinear}\right\|_2 \tag{9}$$

Updating the parameters can be done with any suitable minimization algorithm.

3. SIMULATIONS

The method described previously will be illustrated by an example where parameter estimation is performed on a stick-slip system. Two systems are used in the simulation, one reference system and one estimation system. Both systems are shown in figure 3.1.

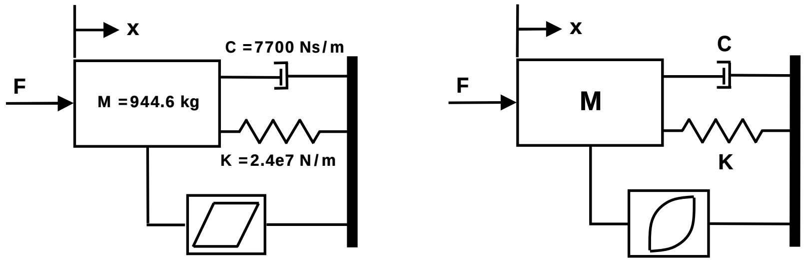

Figure 3.1: The systems used in the simulation. The system to the left is used as reference system, i.e. the system the simulated measurements are performed on. The system to the right is used as estimation system, i.e. the system the parameter estimation is performed on.

The hysteresis loop in the estimation system is governed by equation (10):

$$F = \frac{k_d \cdot x}{\left(1 + \left(\frac{k_d \cdot x}{F_d}\right)^N\right)^{\frac{1}{N}}} \tag{10}$$

k_d - The stiffness during stick-condition

F_d - The force level where the system starts to slip

N - Determines the curvature

The exponent N in equation (10) makes the hysteresis loop far more adaptable than a normal stick-slip function and thereby more suitable for an arbitrary hysteresis function, which is the reason this function was chosen to be used in the estimation process. Figure 3.2. shows hysteresis loops obtained at the same force level but with different values on N.

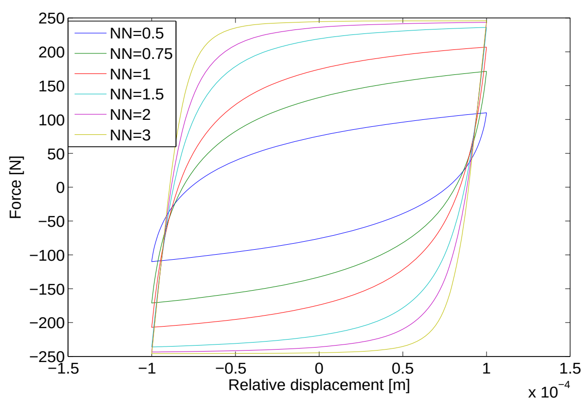

Figure 3.2: Hysteresis loops obtained at the same force amplitude with different values of N, equation (10)

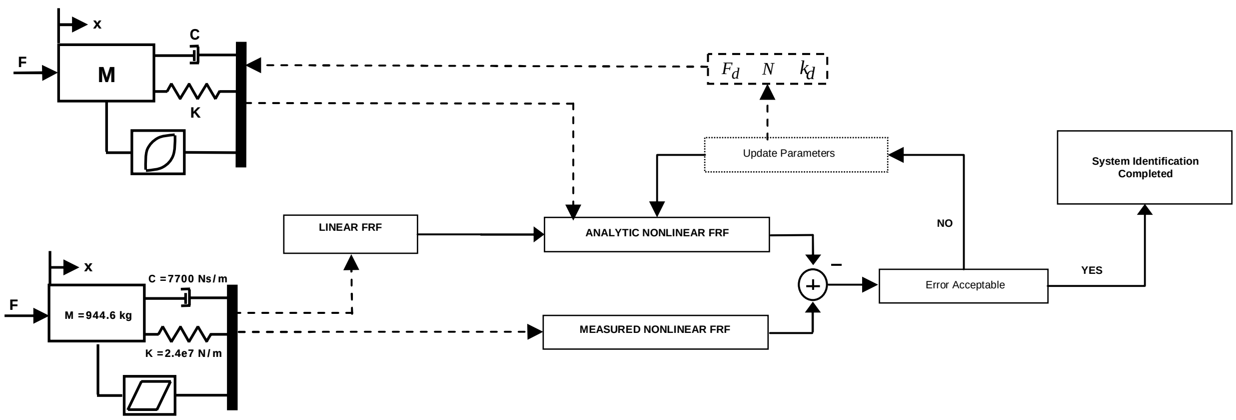

Figure 3.3: Flowchart of the estimation process showing where the two different systems have been used.

The simulations were carried out with 10% noise added to all response signals. The underlying linear system was estimated by exciting the reference system with a large sinusoidal force. The initial guess of k_d was estimated by taking the difference in stiffness of the underlying linear system and the stiffness of the reference system at low force levels. k_d, F_d and N were then estimated according to the flowchart in figure 3.3 using the MATLAB function *fminsearch*. All time responses were simulated by a digital filter approach, see [1]. The results of the simulation are displayed in table 3.1 and figure 3.4.

	F_d	k_d	N
Reference Parameters	246.048 N	2.4e7 N/m	
Estimated Parameters	242.05 N	2.33e7 N/m	22.55

Table 3.1: Reference and estimated parameters.

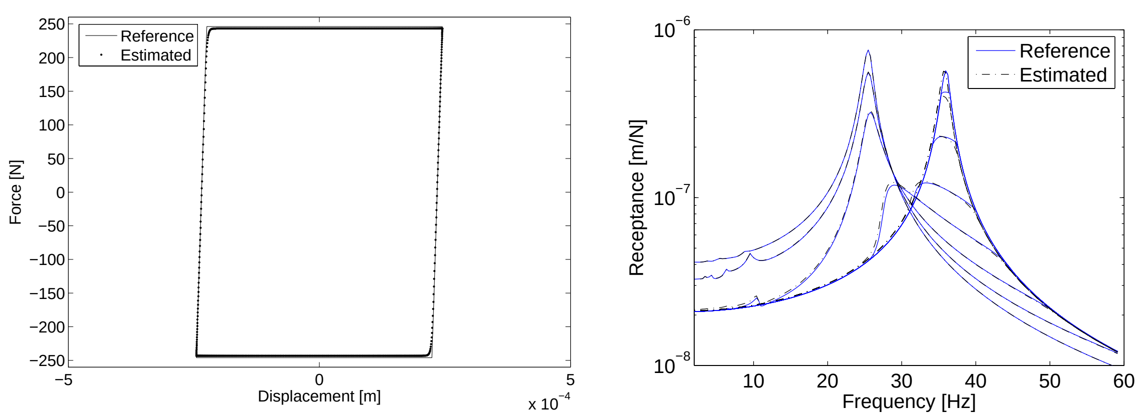

Figure 3.4: Results of the parameter estimation – The figure to the left shows a comparison between a reference and an estimated hysteresis loop. The right figure compares the FRFs obtained from the reference and estimated system by sinusoidal sweeps at different force amplitudes.

As can be seen in figure 3.4 the estimated system has a close match with the reference system over the entire force range. For a more detailed description of the simulation see [2].

4. EXPERIMENTAL TEST

The experimental test rig chosen consists of two cantilever beams connected by a lap joint. This is the same rig as used by Auman et.al. [3].The system, shown in figure 4.1, dissipates energy through microslip at the joint interfaces and therefore shows hysteresis behaviour. The mathematical model of the hysteresis function used in this chapter is shown in equation (10).

Figure 4.1: The experimental test rig.

The reason for choosing this system is to make an initial test of the proposed parameter estimation method, not to make a thorough investigation of the friction phenomenon in joints. Figure 4.2 shows a set of FRFs obtained from sinusoidal sweeps at different force levels. This set of FRFs clearly shows we are dealing with a hysteresis nonlinearity.

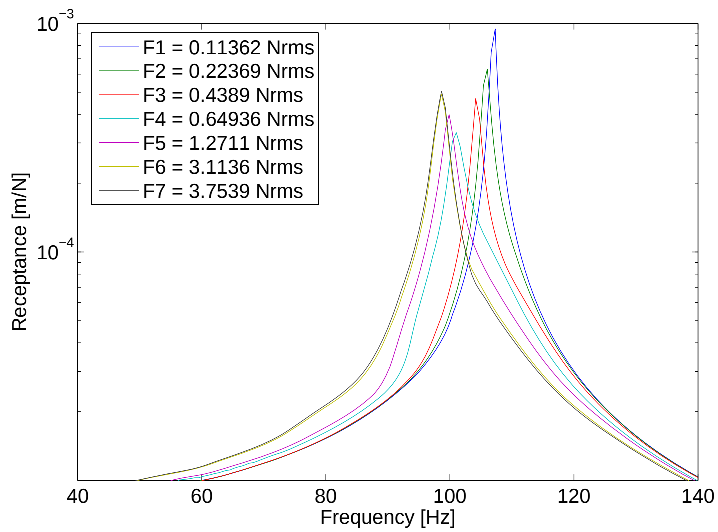

Figure 4.2: Frequency response functions obtained by sinusoidal sweeps at different force amplitudes.

At low levels of excitation the system have low damping and a high resonance frequency. The hysteresis function is considered to be in constant stick at this excitation level and therefore the FRF is considered to be linear. At high levels of excitation the FRF reaches at a certain frequency and amplitude and remains there as the excitation force increases. The same frequency and amplitude is reached regardless of the clamping pressure in the joint, assumed that it's possible to excite the system with the required force. This FRF is considered to be the best approximation of the underlying linear system that can be obtained by measurements. However, the approximation isn't good enough to be used in the parameter estimation process. Because of the high damping in this system the response amplitude at low forces will be impossible to predict accurately. Therefore the damping in this model has to be adjusted. By exciting the system at low force amplitudes it will behave linearly and thereby it is possible to estimate both the stiffness during stick and damping of the system. Having adjusted the estimation of the underlying linear system and the stiffness during stick, k_d, it was now possible to estimate the remaining parameters F_d and N of the hysteresis loop. By picking one of the nonlinear FRFs shown in figure 4.2 an estimation of F_d and N is obtained by fitting the analytical FRF calculated by harmonic balance to the nonlinear FRF. Since the presence of higher harmonics in the measurements and in simulations was small, only the fundamental harmonic was used when calculating the analytical FRF. The results of the estimation processes are displayed in figure 4.3 and 4.4.

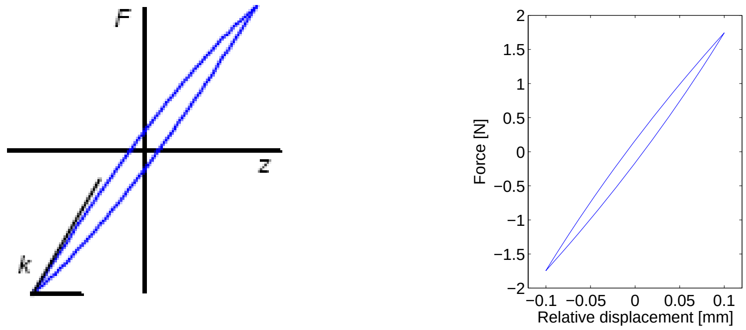

Figure 4.3: The left figure shows an example of a hysteresis loop for a mechanical joint interface obtained from experimental measurements by Smallwood et.al [4]. The right figure shows the hysteresis loop estimated by the method proposed in this paper.

As seen in figure 4.3 the estimated hysteresis loop is very similar to the expected hysteresis loop of a mechanical joint.

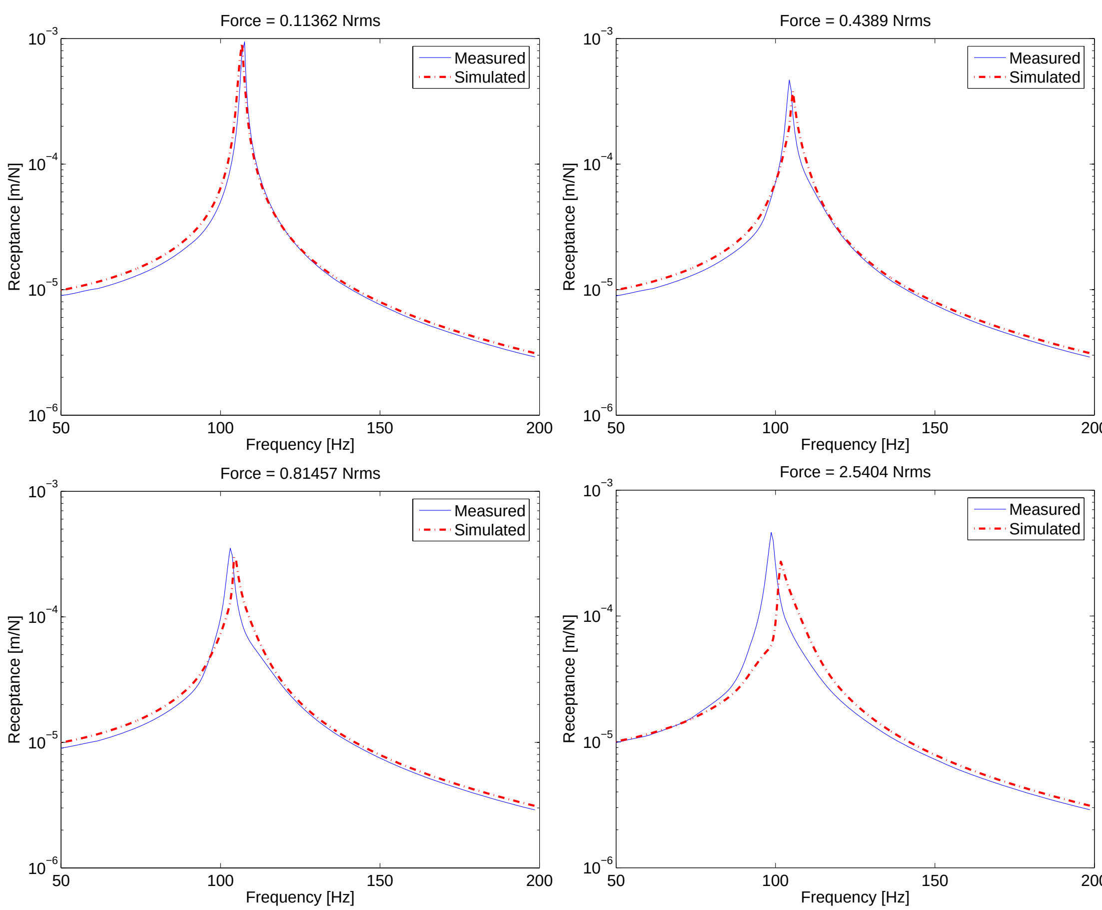

Figure 4.4: Comparison of measured FRFs and FRFs obtained by simulations using the same input force as in the measurements.

Figure 4.4 shows that the theoretical model is able to predict the FRF of the true system within a small error margin at low force levels. But at high force levels the difference between the simulated and true FRF becomes too large, both in frequency and amplitude for the prediction to be considered valid. Figure 4.5 shows the measured response and the simulated response of the structure at high force levels. It is evident that there is something in the structure that is not taken into account in the theoretical model. So modeling the joint as a simple SDOF system is not enough to predict the complete behavior.

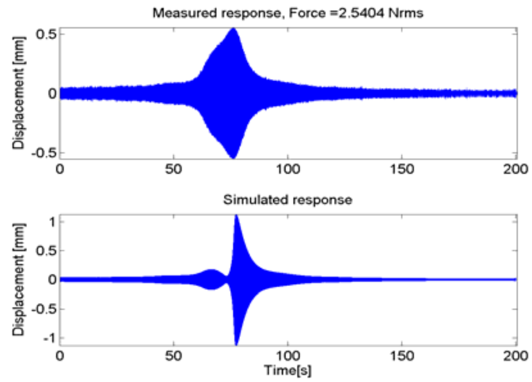

Figure 4.5: Measured and simulated time responses at high force levels.

5. CONCLUSIONS AND FUTURE WORK

A method of parameter estimation for hysteresis elements using harmonic input has been presented and tested in both simulations and on an experimental test rig.

The result from simulations looks very promising. Even though 10 % noise was added to all response signals the estimated system was able to predict the FRFs of the reference system over the entire force range used in the simulations.

The results from the experimental test rig are not conclusive. The estimated hysteresis loop looks exactly as expected and it was possible to estimate the FRF of the system at low excitation levels. But the estimated system was unable to predict the FRF of the true system at high levels of excitation. Therefore additional tests have to be made on well defined hysteresis systems before any conclusions of the functionality of the proposed algorithm can be made.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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